Physics 108 Homework 9

1. a) The Schwarzschild radius is

$$R = \frac{2GM}{c^2} = \frac{2G(4\times10^6 M_\odot)}{c^2}$$

$$= 4\times10^6 \; \frac{2GM_\odot}{c^2}$$

and we know that for the Sun

$$\frac{2GM_\odot}{c^2} = 3\,km$$

So

+5 $$R = 4\times10^6 \times 3\times10^3 m = 1.2\times10^{10} \text{ meters}$$

b)

+5 $$R = 1.2\times10^{10} \cancel{m} \times \frac{1\,AU}{1.5\times10^{11} \cancel{m}} = 0.08\,AU$$

Let's compare the size of the black hole to the radius of the sun, $R_\odot = 6.960\times10^8\,m$

$$\frac{1.2\times10^{10}\,m}{6.960\times10^8\,m} = 17 \text{ times the radius of the Sun}$$

(only that large)

2. a)

+5 $7.0 \times 10^{12}\,eV \times \frac{1.6\times10^{-19}\,J}{1\,eV} = 1.12 \times 10^{-6}\,J$

b) $E = mc^2$ so $m = E/c^2$

+5

$m = \frac{1.12\times10^{-6}\,J}{(2.9979\times10^{8}\,m/s)^2} = 1.246\times10^{-23}\,kg$

+5 c) $R = \frac{2GM}{c^2} = \frac{2(6.67\times10^{-11}\,N\cdot m^2/kg^2)(1.246\times10^{-23}\,kg)}{(2.9979\times10^{8}\,m/s)^2}$

$= 1.85\times10^{-50}\,m$

+3 d) The Schwarzschild radius is smaller than the Planck length

+2 e) Don't worry.

+3 f) At twice the collision energy the Schwarzschild radius will be twice as large, but still much smaller than the Planck length. Still no worries.

3, Data can be obtained from kepler.nasa.gov. You can be optimistic or pessimistic, and I'll try both. To be optimistic, choose the higher value for a numerator and lower value for a denominator.

a) Kepler has been observing between 100,000 and 150,000 stars. There are only 61 confirmed planets (pessimist) but there 1790 host stars with planet candidates.

+5

Optimist: $f_p = \dfrac{1790}{100,000} = 1.79 \times 10^{-2}$

Pessimist: $f_p = \dfrac{61}{150,000} = 4.0 \times 10^{-4}$

Any value between these two is "reasonable"

b) Only one planet, Kepler 22b, has been found in a habitable zone. Thus

+5

Optimist: $n_e = \dfrac{1}{61} = 1.64 \times 10^{-2}$

pessimist: $n_e = \dfrac{1}{1790} = 5.6 \times 10^{-4}$

Note that the optimist and pessimist have swapped numbers for the planet count, now that it's in the denominator. That's not strictly honest, but we are just estimating the high and low values, so go with it.

c) Drake's original values, from the slide shown in class, were

$R^* = 10/yr$

$f_\ell = 0.5$

$f_i = 0.01$

$f_c = 0.01$

$L = 10{,}000$ yr

+5

Resulting in

Optimist:

$$N = (10)(1.79\times10^{-2})(1.64\times10^{-2})(0.5)(0.01)(0.01)(10{,}000)$$

$$= 1.47\times10^{-3}$$

Pessimist:

$$N = (10)(4\times10^{-4})(5.6\times10^{-4})(0.5)(0.01)(0.01)(10{,}000)$$

$$= 1.12\times10^{-6}$$

48

Neither result is encouraging, but remember Kepler will keep searching and find more planets and N will then increase.